Learning with Hidden Information Hung-yi Lee

Different Kinds of Learning

- Supervised Learning
 - Data: $\{(x^1, \hat{y}^1), (x^2, \hat{y}^2), \dots\}$
- Semi-supervised Learning
 - Data: { $(x^1, \hat{y}^1), (x^2, \hat{y}^2), \dots, (x^{N+1}, ?), (x^{N+2}, ?) \dots$ }
- Unsupervised Learning
 - Data: $\{(x^1,?), (x^2,?), \dots\}$
- Hidden variable learning

• Data:
$$\{(x^1, \hat{y}^1), (x^2, \hat{y}^2), \dots\}$$

Some useful information is hidden.





Example Applications for Hidden Variable Learning

Sentiment Analysis: Automatically identify a movie review is positive or negative

Collecting documents about reviewing movies



Sentiment Analysis: Automatically identify a movie review is positive or negative

Filter out the irrelevant part



Only part of the document is related to movie review

Which parts are related to movie is hidden information.

Summarization: Given a long document, select a set of sentences to form a compact version



Speech Recognition





phonemes/states and acoustic features is hidden.

En

Machine Translation

What is the anticipated cost of collecting fees under the new proposal ?

vertu de les nouvelles propositions quel est le coût prévu de perception de les droits 17

The word alignment of the sentence pairs is hidden.

https://buffy.eecs.berkeley.edu/PHP/r esabs/resabs.php?f_year=2006&f_su bmit=chapgrp&f_chapter=12

English

French

There is a general framework.

Two Steps, Three Questions



Two Steps

Step 1: Training

- Find function F
 - $F: X \times Y \times H \to R$
- F(x, y, h) evaluate how compatible x, y and h is

Step2: Inference (Testing)

- Given object x
 - $\tilde{y} = \arg \max_{y} \max_{h} F(x, y, h)$
 - $\tilde{y} = \arg \max_{y} \sum_{h} F(x, y, h)$

Which one is more reasonable?

Three Problems

- Problem 1: Evaluation
 - What does **F**(**x**, **y**, **h**) look like?
 - E.g. $F(x, y, h) = w \cdot \Psi(x, y, h)$
- Problem 2: Inference
 - $\tilde{y} = \arg \max_{y} \max_{h} F(x, y, h)$
 - $\tilde{y} = \arg \max_{y} \sum_{h} F(x, y, h)$
- Problem 3: Training
 - Given { $(x^1, \hat{y}^1), \cdots, (x^n, \hat{y}^n), \cdots, (x^N, \hat{y}^N)$ }
 - EM-like algorithm

Three Problems - Training

Given Training data: $\{(x^1, \hat{y}^1), \cdots, (x^n, \hat{y}^n), \cdots, (x^N, \hat{y}^N)\}$



Taking object detection as Example

Motivation

• An object can have more than one types

Train

Bicycle







Haruhi









short hair

long hair

Motivation



Type1. Short hair



Type 2. Long hair

OriginalFor $\forall y \neq \hat{y}^1$: $w \cdot \phi(x^1, \hat{y}^1) > w \cdot \phi(x^1, y)$ Training:For $\forall y \neq \hat{y}^2$: $w \cdot \phi(x^2, \hat{y}^2) > w \cdot \phi(x^2, y)$

➢ Because φ(x¹, ŷ¹) and φ(x², ŷ²) can be very different
 ➢ It may be hard to use a single w to achieve the above goal

Two Cases

- Involving object types into object detection
- <u>Case 1</u>
 - The useful information is available on training data, only hidden in testing data
 - Not too much difference from original structured SVM, extra efforts for labelling

• <u>Case 2</u>

- The information is hidden in both training and testing data
- What we really care about

Case 1: Two kinds of Objects?

 There are two kinds of objects to be detected: Haruhi_1 and Haruhi_2



Haruhi_1



Haruhi_1



Haruhi_2



Haruhi_2



Haruhi_1



Haruhi_1



Haruhi 2



Haruhi_2

Case 1: Two kinds of Objects? Haruhi_1 Haruhi_2



Evaluation:

$$F_1(x,y) = w_1 \cdot \phi(x,y)$$

Training Target:

 x^n is Haruhi_1

 $w_1 \cdot \phi(x^n, \hat{y}^n) > w_1 \cdot \phi(x^n, y)$





Evaluation:

$$F_2(x,y) = w_2 \cdot \phi(x,y)$$

Training Target: x^n is Haruhi 2 $w_2 \cdot \phi(x^n, \hat{y}^n) > w_2 \cdot \phi(x^n, y)$

- Now we have w_1 for Haruhi_1 and w_2 for Haruhi_2
- Inference:

Given an image x



If the Harihu in image is Haruhi_1: $\tilde{y}_1 = \arg \max_{y \in \mathbb{Y}} w_1 \cdot \phi(x,y)$ If the Harihu in image is Haruhi_2: $\tilde{y}_2 = \arg \max_{y \in \mathbb{Y}} w_2 \cdot \phi(x,y)$

Critical Problem: Given an input image, we do not know the Haruhi in the image is Haruhi_1 or Haruhi_2

• Inference



Input

If we know its type 1

We don't know the type of the input image actually.



• w_1 and w_2 are learned separately

Training Target: x^n is Haruhi 1 $w_1 \cdot \phi(x^n, \hat{y}^n) > w_1 \cdot \phi(x^n, y)$ 0.09 0.1 **Training Target:** x^n is Haruhi 2 $w_2 \cdot \phi(x^n, \hat{y}^n) > w_2 \cdot \phi(x^n, y)$ 1000000 999999 $x^{n} \text{ is Haruhi_1}$ $w_{2} \cdot \phi(x^{n}, y)$ v $w_{1} \cdot \phi(x^{n}, \hat{y}^{n})$ v $w_{1} \cdot \phi(x^{n}, y)$

w₁ and w₂ should be learned jointly

Case 1: Evaluation

For "type 1",
$$F(x, y) = w_1 \cdot \phi(x, y)$$

For "type 2", $F(x, y) = w_2 \cdot \phi(x, y)$

$$F(x, y, h) = w \cdot \Psi(x, y, h)$$

h: type of Haruhi (type 1 or type 2) $\Psi(x, y, h)$: a feature vector for x, y and type h Its length is twice of $\phi(x, y)$ w: a weight vector to be learned Its length is twice of w_1 or w_2

Case 1: Evaluation

$$F(x, y, h) = w \cdot \Psi(x, y, h)$$

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \begin{cases} \Psi(x, y, h = "type \ 1") = \begin{bmatrix} \phi(x, y) \\ 0 \end{bmatrix} \\ \Psi(x, y, h = "type \ 2") = \begin{bmatrix} 0 \\ \phi(x, y) \end{bmatrix}$$

For "type 1", $F(x, y, h) = w_1 \cdot \phi(x, y) + w_2 \cdot 0$ For "type 2", $F(x, y, h) = w_1 \cdot 0 + w_2 \cdot \phi(x, y)$

Case 1: Inference

ŷ $arg \max_{y} \max_{h} w \cdot \Psi(x, y, h)$



$$\tilde{y} = \arg \max_{y} w \cdot \phi(x, y)$$

$$C^{n} = \max_{y} [w \cdot \phi(x^{n}, y)] - w \cdot \phi(x^{n}, \hat{y}^{n})$$

$$C^{n} = \max_{y} [\Delta(\hat{y}^{n}, y) + w \cdot \phi(x^{n}, y)] - w \cdot \phi(x^{n}, \hat{y}^{n})$$

$$\tilde{y} = \arg \max_{y} \max_{h} w \cdot \Psi(x, y, h)$$

$$C^{n} = \max_{y} \max_{h} [w \cdot \Psi(x^{n}, y, h)] - w \cdot \Psi(x^{n}, \hat{y}^{n}, \hat{h}^{n})$$

$$C^{n} = \max_{y} \max_{h} [\Delta(\hat{y}^{n}, y) + w \cdot \Psi(x^{n}, y, h)] - w \cdot \Psi(x^{n}, \hat{y}^{n}, \hat{h}^{n})$$





Given training data: $\{(x^1, \hat{y}^1, \hat{h}^1), \dots, (x^n, \hat{y}^n, \hat{h}^n), \dots, (x^N, \hat{y}^N, \hat{h}^N)\}$

$$C^{n} = \max_{y} \max_{h} [\Delta(\hat{y}^{n}, y) + w \cdot \Psi(x^{n}, y, h)] -w \cdot \Psi(x^{n}, \hat{y}^{n}, \hat{h}^{n})$$

Find $w, \varepsilon^{1}, \dots, \varepsilon^{n}, \dots, \varepsilon^{N}$ minimize: $\frac{1}{2} ||w||^{2} + \lambda \sum_{n=1}^{N} \varepsilon^{n}$
 $\forall n, \forall y \in \mathbb{Y}, \forall h \in \mathbb{H}$
 $w \cdot \Psi(x^{n}, \hat{y}^{n}, \hat{h}^{n}) - w \cdot \Psi(x^{n}, y, h) \ge \Delta(\hat{y}^{n}, y) - \varepsilon^{n}$

• The useful information are usually *hidden*



Type 1



Type 1



Type 2



Type 2



Type 1



Type 1



Type 2



Type 2

How to deal with hidden information with Structured SVM?

No types? Try to generate ourselves





$$(x^{3}, \hat{y}^{3})$$





$$F(x, y, h) = w \cdot \Psi(x, y, h)$$

Random initialized
$$w = w^{0}$$

Evaluate the compatibility of x, y and h

$$\tilde{h} = \arg \max_{h} w \cdot \Psi(x, \hat{y}, h)$$

Given x and \hat{y} , find the most compatible h

No types? Try to generate ourselves











 \tilde{h}^1 = type 1 \tilde{h}^2 = type 2 \tilde{h}^3 = type 1



$$\overline{\tilde{h}^4}$$
= type 2

For n = 1, ..., 4: $\tilde{h}^n = \arg \max_{h} w^0 \cdot \Psi(x^n, \hat{y}^n, h)$

Good guess? Of course not.

Because w0 is random

 (x^{3}, \hat{y}^{3})

 (x^{4}, \hat{y}^{4})

• With the types we generate, we can find a w

 (x^1, \hat{y}^1) (x^2, \hat{y}^2)



Case 2: Training with Hidden Information For n = 1, ..., 4: $\tilde{h}^n = \arg \max w^1 \cdot \Psi(x^n, \hat{y}^n, h)$ (x^2, \hat{y}^2) (x^{3}, \hat{y}^{3}) (x^4, \hat{y}^4) (x^1, \hat{y}^1) \tilde{h}^1 = type 1 \tilde{h}^2 = type 2 \tilde{h}^3 = type 2 \tilde{h}^4 = type 2 w^1 Solving a QP

Is w^1 a good weight vector? Probably not

Train from random $ilde{h}$

Case 2: Training with Hidden Information For n = 1, ..., 4: $\tilde{h}^n = \arg \max_h w^2 \cdot \Psi(x^n, \hat{y}^n, h)$



Is w^2 better than w^1 ? Yes (?)

Iteratively



Why we can get better weight vector after each iteration?

Warning of Math



Structured SVM

Training data: $\{(x^1, \hat{y}^1), \cdots, (x^n, \hat{y}^n), \cdots, (x^N, \hat{y}^N)\}$

Minimizing cost

$$\tilde{y} = \arg \max_{y} w \cdot \phi(x, y)$$

$$C = \frac{1}{2} \|w\|^{2} + \sum_{n=1}^{N} C^{n} \geq \sum_{n=1}^{N} \Delta(\hat{y}^{n}, \tilde{y}^{n})$$

$$C^{n} \geq \Delta(\hat{y}^{n}, \tilde{y}^{n})$$

$$C^{n} = \max_{y} [\Delta(\hat{y}^{n}, y) + w \cdot \phi(x^{n}, y)] - w \cdot \phi(x^{n}, \hat{y}^{n})$$

What does the function Cⁿ look like?

Structured SVM







Structured SVM

There is no local minima for structured SVM.







Training data: $\{(x^1, \hat{y}^1), \cdots, (x^n, \hat{y}^n), \cdots, (x^N, \hat{y}^N)\}$

In each iteration, the $\tilde{y} = \arg\max_{v} \max_{h} w \cdot \Psi(x, y, h)$ following cost is smaller $C = \frac{1}{2} \|w\|^2 + \sum_{n=1}^{N} C^n \geq \Delta(\hat{y}^n, \tilde{y}^n) \sum_{n=1}^{N} \Delta(\hat{y}^n, \tilde{y}^n)$ $C^{n} = \max_{y} \max_{h} [\Delta(\hat{y}^{n}, y) + w \cdot \Psi(x^{n}, y, h)]$ $-\max_{h} w \cdot \Psi(x^n, \hat{y}^n, h)$





Cost function to be minimized

$$C^{n} = \max_{y} \max_{h} [\Delta(\hat{y}^{n}, y) + w \cdot \Psi(x^{n}, y, h)] - \max_{h} w \cdot \Psi(x^{n}, \hat{y}^{n}, h)$$
convex
convex

Cost function to be minimized













$$\tilde{h}^{n} = \arg \max_{h} w \cdot \Psi(x^{n}, \hat{y}^{n}, h)$$

$$\underbrace{w^{0}}_{\text{to form the auxiliary function}} W$$

$$to form the auxiliary function$$

$$C^{n} = \max_{y} \max_{h} [\Delta(\hat{y}^{n}, y) + w \cdot \Psi(x^{n}, y, h)] - \max_{h} w \cdot \Psi(x^{n}, \hat{y}^{n}, h)$$

$$\underbrace{convex}_{\text{concave}} \tilde{h}^{n} = \arg \max_{h} w^{0} \cdot \Psi(x^{n}, \hat{y}^{n}, h)$$

$$A(w) = \max_{y} \max_{h} [\Delta(\hat{y}^{n}, y) + w \cdot \Psi(x^{n}, y, h)] - w \cdot \Psi(x^{n}, \hat{y}^{n}, \tilde{h}^{n})$$

$$\underbrace{Minimizing A(w)}_{\text{Minimizing A(w)}} W$$

$$A(w) = \max_{y} \max_{h} [\Delta(\hat{y}^{n}, y) + w \cdot \Psi(x^{n}, y, h)] - w \cdot \Psi(x^{n}, \hat{y}^{n}, \tilde{h}^{n})$$

find the minimum value
Solving a QP
$$A(w) = \max_{y} \max_{h} [\Delta(\hat{y}^{n}, y) + w \cdot \Psi(x^{n}, y, h)] - w \cdot \Psi(x^{n}, \hat{y}^{n}, \tilde{h}^{n})$$
$$w \cdot \Psi(x^{n}, \hat{y}^{n}, \tilde{h}^{n}) - \max_{y} \max_{h} [\Delta(\hat{y}^{n}, y) + w \cdot \Psi(x^{n}, y, h)] = -A(w)$$
$$\forall y \in \mathbb{Y}, \forall h \in \mathbb{H}$$
$$w \cdot \Psi(x^{n}, \hat{y}^{n}, \tilde{h}^{n}) - [w \cdot \Psi(x^{n}, y, h) + \Delta(\hat{y}^{n}, y)] \ge -A(w)$$
$$\forall y \in \mathbb{Y}, \forall h \in \mathbb{H}$$
$$w \cdot \Psi(x^{n}, \hat{y}^{n}, \tilde{h}^{n}) - w \cdot \Psi(x^{n}, y, h) \ge \Delta(\hat{y}^{n}, y) - A(w)$$

End of Warning





To Learn More ...

- Framework
 - Chun-Nam John Yu and Thorsten Joachims, "Learning Structural SVMs with Latent Variables," ICML 2009
- Video
 - Wang, Yang, and Greg Mori. "Max-margin hidden conditional random fields for human action recognition," CVPR 2009
 - Wang, Yang, and Greg Mori. "Hidden part models for human action recognition: Probabilistic versus max margin," *Pattern Analysis and Machine Intelligence, IEEE Transactions,* 2011
- Image
 - Zhu, Long, et al. "Latent hierarchical structural learning for object detection."*Computer Vision and Pattern Recognition (CVPR), 2010 IEEE Conference on*. IEEE, 2010.
 - Felzenszwalb, Pedro F., et al. "Object detection with discriminatively trained part-based models." *Pattern Analysis and Machine Intelligence, IEEE Transactions on* 32.9 (2010): 1627-1645.
- Language processing
 - Sun, Xu, et al. "Latent Variable Perceptron Algorithm for Structured Classification," *IJCAI*. Vol. 9. 2009
 - http://speech.ee.ntu.edu.tw/~tlkagk/courses/MLDS_2015/Structured%20Le cture/Summarization%20Hidden_2.ecm.mp4/index.html

Appendix: EM in one slide



EM in one slide



